

Nom et Prénom : EL HILALI ABDENBI

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Directeur de Thèse : EL WAHBI BOUAZZA

Sujet de Thèse :

On a study of properties of certain entropy operators via some extended metrics to positive definite matrices

Abstract:

This thesis is centered on the study of certain properties of a class of parametric relative entropy and Tsallis entropy operators acting on positive definite matrices. The study aims to determine certain geometric characteristics of these operators. Precisely, by taking into account certain distances and divergences, we were able to estimate distances involving some operator entropies.

The choice of this theme was motivated by the importance of the notion of entropy as a tool for studying some dynamic systems relevant to several disciplinary fields. This importance has led to extensions of this concept, initially defined for real numbers, to matrices and then to operators defined on a Hilbert space and very recently to convex functions.

The central question in this work is the study of certain geometric properties of the parametric entropy operators defined by Constantino Tsallis in 1988 and those introduced by Takayuki Furuta in 2004. Given a parameterized operator entropy Op and a certain distance or divergence d , the main questions in this work are articulated around the following three axis.

1. The comparison of the following two values : $d(A, Op(A, B))$ and $d(A, B)$.
2. The variations of the map $p \mapsto d(A, Op(A, B))$.
3. Estimation of distance between a parametric operator entropy and some operator means in the sense of Kubo-Ando.

Where A and B stand for positive definite matrices. Three metric types are evoked in this work. The Riemannian and log-determinant distances which are useful in several areas as information theory for instance. The third one concerns some versions of Hellinger distance defined very recently.

Regarding the theoretical framework used, the investigations are based on tools of matrix analysis and the theory of means known in the literature by the Kubo-Ando theory. Interesting results were obtained. They were generally expressed by inequalities which translate estimations of distances involving the entropy operators and which make it possible to locate in different ways the entropy of two positive definite matrices. In particular, we establish that the entropy of two positive definite matrices lies in a sphere of radius the half of the distance between these matrices.

Keywords: relative operator entropy, Tsallis relative operator entropy, positive definite matrices, metrics, matrix inequalities, means