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Sujet de Thèse :

Some developments in frame theory

Abstract :

In recent years, there has been a rapid growth of interest in fame theory, as well as its versatile applications. A recent development in frame theory leads to the introduction of the concept of Probabilistic frame in [16, 17] and was further expanded in [18]. The probabilistic frames can be seen as a generalization of frames and as a subclass of continuous frames. The present thesis is concerned with frames in a probabilistic setting.

In Chapter 1, we survey relevant background mathematics from frame theory, probability theory and Positive Operators Valued Measures which we use through-out the thesis.

In Chapter 2, we investigate how far is the closest probabilistic tight frame from a given probability measure where the distance used is the quadratic Wasserstein metric W2 used for optimal transportation problem for measures.

In particular, we study the optimization problem

$$I(\mu;K) := \inf_{\nu \in T(K)} W_2^2(\mu; \nu);$$

where T (K) is the set of all probabilistic tight frames whose supports are contained in $K = R^n$ or $K = S^{n-1}$. We solve this problem in the general case, and in addition, we establish the uniqueness of the optimum and we give its explicit expression. In the other case, we give concise upper and lower boundsfor $I(\mu; S^{n-1})$.

In Chapter 3, we solve an open problem stated in [18] related to the representation of Positive Operators Valued Measures by means of tight probabilistic frames.

In Chapter 4, we study frames from the viewpoint of coding theory. We introduce a new probabilistic model that is robust when erasures occurring in data transmission using Parseval frames and a sequence of Bernoulli random variables associated to the channels of the transmission. We establish several results characterizing the optimal Parseval frames for our model. It is shown that our model is more effective than existing models [6, 24, 31, 33] on recovering the signal.

In the Appendix A, we present a new algorithm whose purpose is to construct finite frames with a given frame operator.

KEY WORDS:
Frames, probabilistic frames, tight probabilistic frames, optimmal transport, wesserstein metric, Parseval frames, Tight frames, dual frames, erasures